Infrared Range Diffraction Radiation from Two Dielectric Rods Covered with Graphene as a Tool for Sensing the Charged-Particle Beam Position

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Abstract This work presents a research into the infrared range diffraction radiation (DR) of a modulated beam of charged particles moving between twin dielectric circular nanowires covered with graphene. The latter are assumed zero-thickness and characterized with the Kubo formalism and resistive-type boundary conditions. We assume that the beam velocity is constant and apply the separation of variables in the local coordinates, combined with the addition theorem for the cylindrical functions. This enables us to cast the problem of DR to a Fredholm second-kind matrix equation and, therefore, guarantees the convergence. Due to convergence, both far and near DR-field characteristics can be computed with a controlled accuracy. The analysis shows the appearance of additional resonances linked with a deviation of the beam trajectory from the symmetric position. “Shining” of these resonances can be viewed as an instrument for the beam position monitoring with a nanoscale antenna.

Index Terms — diffraction radiation, electron beam, graphene, plasmon resonance, infrared waves, beam position monitor

I. INTRODUCTION

Graphene is a monolayer or a thin stack of parallel layers of graphite and hence has a sub-nanometer thickness. Among its outstanding properties, there is visible range transparency, great mechanical strength, and good electron conductivity at the terahertz (THz) and infrared (IR) frequencies. The graphene conductivity is a function of the electron relaxation time, temperature, chemical doping and frequency. This makes graphene’s electromagnetic properties close to those of the noble metals in the visible-light range, however at much lower frequencies. Namely, graphene supports the plasmon guided wave at the THz and IR frequencies that is tunable with the DC bias, which transmits to the chemical potential [1,2]. Although graphene is more often attached to flat substrates made of dielectric, curved substrates are attracting a growing attention today [3]. This opens new opportunities for the design of micro and nanoscale tunable scatterers of the THz and IR waves.

Diffraction Radiation (DR) is the effect of electromagnetic-wave emission, which accompanies the charged particles passing near dielectric and metallic objects. The Smith-Purcell effect [4] is the earliest example of DR in the form of the visible light radiation from the beam of charged particles flying above a grating across its grooves. It was theoretically analyzed in [5-8] and other works. Among the applications of DR, important place is occupied by the design of non-invasive beam position monitors (BPM) for accelerators and colliders. Here, existing DR-based BPMs work in the microwave range [9-14] and are somewhat bulky. Therefore, a shift to the optical DR with BPMs based on the IR, visible-light, and ultraviolet emission is extremely attractive. Such a shift has become possible thanks to the availability of the nanoscale circuit elements manufactured with the aid of the modern nanotechnologies [15-20].

In the DR analysis, the beam velocity and trajectory are viewed as fixed. Then, one can neglect the deceleration of the beam and use the linear electromagnetic formulation, i.e. consider DR modelling as a traditional wave-scattering problem. In terms of physics, the main difference comes from the fact that the incident field is a field of charged particles or a beam in the free space instead of more common spherical or plane waves. Such a field is a slow wave traveling with the beam velocity.

In order to monitor the parameters of electron beam, one should measure the intensity of DR in the far or near zone. To raise the sensitivity of BPM, it is necessary to find an advantageous combination of its configuration and materials of its elements. Here, a promising option is the use of resonance effects because a resonance increases the DR intensity as a square of its Q-factor. This is a common case in the microwaves when the variety of metallic hollow cavities are integrated coaxially in the drift tubes. To reduce the BPM size, they work on the lowest natural modes. This idea can be extended to the THz and IR ranges provided that appropriate sub-wavelength resonators are exploited. One feasible approach is the use of materials with high refractive indices. However, refractive indices of the modern dielectric materials are in the tens of units that means that the lowest mode resonances entail just half-wavelength (in material) dimensions [19]. A remedy can be seen in working with the plasmon modes on the graphene-coated or patterned-graphene scatterers. It should be noted that such configurations are already considered as promising components of sensors of the tunable environment refractive index [21], microsize reflectors [22], antennas, and scatterers [23,24].
The graphene plasmons in IR and THz ranges have the Q-factors in the range of 20 to 100 that is still higher than of a solid metal wire in the visible range. This tells that graphene-coated dielectric rods are promising as small-size resonance scatterers in many applications [21-24] including optical-DR BPMs.

To emulate real-life 3-D configuration with a 2-D modeling, a model of BPM must contain two identical scatterers on the opposite sides of the beam, such as two edges of a slot [14]. Then a distinction in the angular radiation patterns or in the DR intensities in the lower and upper halfspaces can serve as an indicator of a shift in the beam position. This consideration underlines the analysis of the twin solid dielectric nanowire and twin noble-metal nanotube BPM configurations in [19,20]. Note that a useful insight into the hybrid “supermodes” of the circular dielectric and graphene-covered dimers is found in [25,26].

Here, we consider an infrared-range BPM configuration, made of twin dielectric round wires with graphene covers - see Fig. 1. The derivations follow [19,20] with necessary modifications caused by the graphene coatings. Our goal is to study how the beam position trajectory impacts the DR power, especially the appearance of high-Q plasmon resonances.

II. PROBLEM FORMULATION AND BASIC EQUATIONS

We consider a flat beam of electrons flying in parallel to the x-axis at the distance \( h \). The beam is assumed zero-thickness and infinite in the z-direction. Its velocity is fixed, \( v = \beta c \), where \( c \) is the light velocity and \( \beta < 1 \). The density charge is modulated harmonically in time with the frequency \( \omega \) and amplitude \( \rho_0 \),

\[
\rho = \rho_0 \delta(y-h) \exp[i(kx/\beta - \omega t)],
\]

where \( \delta(y) \) is the Dirac delta function and \( k = \omega/c \) is the free-space wavenumber. Expression (1) can be also viewed as a Fourier-transform of the charge of a single 2-D “particle.” In practice, the periodic modulation of the charge density can be achieved with the aid of external laser illumination or periodic waveguide bunching [17].

Following [5], the field generated by (1) is an H-polarized surface wave propagating along the trajectory of the beam,

\[
H_z^t(x,y) = A \delta(y-h) \exp[i(k_x x/\beta - \omega t)],
\]

where \( q = k y/\beta \), \( r = (1-\beta^2)^{1/2} \), sign() = ±1, \( A \) is a constant, and the dependence on the time is omitted.

Fig. 1 shows the analyzed BPM model, which contains two identical circular wires made of dielectric with graphene coatings (denoted #1 and #2). They have the radius \( a \) and refractive index \( \alpha = \sqrt{\epsilon} \) while the air-gap is \( s \) and the distance between the wire axes is \( L \). We suppose that the beam (1) flies in the +x direction between the wires at the distance \( h < s/2 \) from the midpoint of the air-gap. We choose the Cartesian and the global \( (r,\phi) \) and local \( (r_p,\phi_p) \), \( p = 1,2 \), polar coordinates as shown in Fig. 1.

The formulation of the 2-D wave-scattering problem for the DR magnetic field function involves the Helmholz equation, the resistive conditions at the wire contours, the Sommerfeld radiation condition at infinity, and the condition of local power finiteness. These conditions guarantee the solution uniqueness.

We search for the magnetic field as follows:

\[
H^t = \begin{cases} H^t_{\text{int}(p)} & r_p < a_p, \quad p = 1,2 \\ H_0^t + H_0^t & r : |r_p| > a_p, \quad p = 1,2 \end{cases}
\]

Using the azimuthal Fourier series in the local coordinates, we expand the field inside each nanowire and outside of them (domains (1.1) and (1.2)),

\[
H_{\text{int}(p)}(r, \phi) = \sum_{r_p = 1,2, a_p = m} y_p^{(p)} J_n(k a_p r_p) e^{i m \phi}, \quad r_p < a_p, \quad p = 1,2
\]

and

\[
H^t_0(r, \phi) = \sum_{r_p = 1,2} \sum_{n = -\infty}^{\infty} z_n^{(p)} H_n^{(1)}(k a_p r_p) e^{i n \phi}, \quad r_p > a_p
\]

where \( y_p^{(p)}, z_n^{(p)} \) are unknown coefficients.

In (4) and (5), \( H_n^{(1)} \) and \( J_n \) are the first-kind Hankel and the Bessel functions, respectively.

The resistive-type boundary conditions at the wire contours

\[
E_{\text{int}(p)}^t(r, \phi) = E_{\text{ext}(p)}^t(r, \phi) + \frac{\partial E_{\text{int}(p)}^t(r, \phi)}{\partial n},
\]

where \( E_{\text{int}(p)}^t, E_{\text{ext}(p)}^t \) are the electromagnetic field inside and outside nanowire.

Following [27] and neglecting the wire curvature, the resonance frequencies \( f_m \) of the plasmon modes on a solid circular dielectric wire with graphene cover are found approximately as

\[
f_m \approx (2\pi)^{-1} \left( mc Z_m c \right)^{1/2} [a(\epsilon + 1)]^{1/2},
\]

where \( c_1 \) is a constant that follows from (9) [23,24].

Substituting into (6) and (7) the series (4) and (5) and a model term, which follows from the Kubo formalism [21-24],

\[
\left( \sigma_{\text{int}} \right) = \frac{i q^2 k_p T}{\pi \hbar^2 (\omega + i \tau^{-1})} \left[ \frac{\mu_p}{k_p T} + 2 \ln \left( 1 + e^{-\frac{\omega}{\tau}} \right) \right],
\]

with \( q_e \) is the electron charge, \( k_p \) is the Boltzman constant, \( h \) is the reduced Planck constant, \( T \) is the temperature, \( \tau \) is the electron relaxation time, and \( \mu_p \) is the chemical potential.

As mentioned, graphene-coated wires support the plasmon modes as the standing waves of a 1-D closed contour. They are H-polarized and can be numbered with the index \( m = 1,2,... \) [21]. Following [27] and introducing new unknowns, \( x_n^{(p)} = z_n^{(p)} w_n, \quad w_n = n ![2 / (k a)]^{2m} \),

\[
x_n^{(p)} + V_n D_n^{-1} \sum_{n_m = -n}^{n} (\pm 1)^n w_n H_{n_m - n}(kL) x_m^{(p)} = F_m^{(p)} D_n^{-1}, \quad (11)
\]

\[
V_n = J_n' - iZ \left[ \frac{\alpha J_n'(k a a)}{J_n'(k a a)} - J_n \right],
\]

\[
D_n = w_n \left( H_n' - iZ \left[ \frac{\alpha J_n'(k a a)}{J_n'(k a a)} - H_n \right] \right),
\]

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\[ F_m^{(p)} = iZ \left[ g_m^{(p)}(k \alpha) - g_{-m}^{(p)} \right] - g_m^{(p)} , \]  
\[ g_m^{(1,2)} = \mp A e^{-k(1/2 + h)} m_{n} J_n (1 + \gamma)^{\beta (-\gamma)}, \]  
and the omitted arguments of the cylindrical functions are \( k \alpha \).

The large-argument asymptotic expressions for the Hankel functions enable us to present the scattered field at \( r \to \infty \) as a cylindrical wave, \( H_z^m(r, \phi) = (2 / \pi \alpha k r)^{1/2} \Phi(\phi) \exp(ikr) \), where the scattering pattern is a function of \( \alpha (1,2) \),

\[ \Phi(\phi) = \sum_{n=-\infty}^{\infty} \left( -i \right)^n J_n \left[ e^{-ikz_{m} \sin \phi} z_{m}^{(1)} + e^{ikz_{m} \sin \phi} z_{m}^{(2)} \right] \exp(\alpha), \]  

Then the partial scattering cross-sections (SCS) are

\[ \sigma_{m}^{(1,2)} = \frac{2}{\pi k A^2} \left[ \Phi(\phi) \right]^2 d\phi , \]  

The dielectric wires can be considered lossless, however, this is not true for the graphene covers. Consequently, we represent the partial absorption cross sections (ACS) as

\[ A_m^{(1,2)} = g_m^{(1,2)} e^{ikz_{m} (1/2 + h)} + J_{m} (z) e^{ikz_{m} (1/2 - h)} H_{m} , \]  

It should be noted that the sum of all partial SCS and ACS is the extinction cross-section, \( \sigma_{ext} \), which is associated with the far-field DR pattern amplitude \( (16) \) in the directions of the complex-valued angles of incidence (see \[20\] for details),

\[ \sigma_{ext} = -\frac{4e^{\sqrt{1/2} \alpha}}{k A^2} \left( \sum_{n=-\infty}^{\infty} (-i)^n \beta n J_{n} \left[ e^{ikz_{m} (1/2 + h)} + e^{ikz_{m} (1/2 - h)} \right] \right) , \]  

This expression is the Optical Theorem adapted to the DR effect of a modulated electron beam. In our computations, it has been always fulfilled with machine precision.

### III. NUMERICAL RESULTS

Figs. 2 to 4 present the normalized partial SCS and ACS computed as a function of the frequency for a dimer of twin graphene-coated circular dielectric wires with the radius \( a = 50 \) nm, separated by the air gap of the width \( s = 10 \) nm. Two cases of the beam are considered: not shifted from the midpoint position, \( h = 0 \), and shifted towards the wire #2 by 4 nm.

As can be seen, graphene’s chemical potential \( \mu_c = 0.5 \) eV does not allow to observe clearly an effect of the beam shift. The curves for \( h = 0 \) and \( h = 4 \) nm visually overlap and show only the resonances on the \( y \)-odd plasmon supermodes, \( P_{y,1,2}^{EE} \) and \( P_{y,1,2}^{EO} \), which remain unresolved. This is because the plasmon mode Q-factors of are too low. To rise the Q-factors, one can use the larger chemical potentials or take the wires of smaller radius. Indeed, then a shifted beam SCS and ACS plots show the peaks of new high-Q resonances, which were absent if the beam was not shifted. These new peaks are associated with the unresolved supermodes \( P_{1,2,3}^{EE} \) and \( P_{1,2,3}^{EO} \), which have the field symmetry, orthogonal to the beam field (2) if \( h = 0 \).

![Fig. 2. Normalized partial SCS and ACS versus frequency for twin dielectric nanowires covered by graphene with radius \( a = 50 \) nm, air gap width \( s = 10 \) nm, chemical potential \( \mu_c = 0.5 \) eV, and beam shift \( h = 0 \) and 4 nm.](image1)

![Fig. 3. The same as in Fig.2 for the chemical potential \( \mu_c = 5 \) eV](image2)

![Fig. 4. The same as in Fig.2 for the chemical potential \( \mu_c = 10 \) eV.](image3)

This is exactly the effect, which can be used in the BPM design. Note that its analog was found in the visible-light DR from the beam-excited high-permittivity dielectric nanowire dimers and silver nanotube dimers \[19,20\]. Note that the \( y \)-odd supermode frequencies are higher that the frequency of the \( P_{1,2,3}^{EO} \) plasmons on a stand-alone wire, predicted by \[10\] and shown by the violet dotted lines. In contrast, the frequencies of their \( y \)-even sisters are lower than the single-wire plasmon frequencies. This agrees with the results of \[26\] and with the behavior of the supermodes of dielectric dimers \[25\].

A note should be made that today the best CVD graphene shows the chemical potential of 1 eV, however, higher values can become accessible in not very distant future.

The near fields in Fig. 5, computed in the plasmon-mode resonances for the shifted beam trajectory, show the field variations around the wires that agree with the above presented interpretation.
potential or with smaller values of the nanowire radius. Covered by graphene with radius
Fig. 5. In-resonance near magnetic field patterns of twin dielectric microwires
[37x-206]Foundation of Ukraine, project #2020.02.0150.
[37x-172]can be achieved either with higher values of the chemical
[37x-56]trajectory position between the wires. To be registered easier,
[37x175]supermodes, which are not excited in the case of symmetrical
[37x186]excitation of new high-Q resonances on the hybrid plasmon
[37x313]graphene-covered dielectric nanowires, can monitor a beam
umerical code, an infrared DR based BPM designed of twin
[55x-245]T. Low and P. Avouris, "Graphene plasmonics for terahertz to mid-

IV. CONCLUSION
As shown above with the aid of convergent in-house numerical code, an infrared DR based BPM designed of twin graphene-covered dielectric nanowires, can monitor a beam trajectory shift from a prescribed trajectory. This is due to the excitation of new high-Q resonances on the hybrid plasmon supermodes, which are not excited in the case of symmetrical trajectory position between the wires. To be registered easier, these resonances require higher values of their Q-factors; this can be achieved either with higher values of the chemical potential or with smaller values of the nanowire radius.

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REFERENCES